4-10/2015

II سكشن ٢ م.ع

AMR / M.r

a

Complex Number

$x^2 - 9 = (x-3)(x+3)$

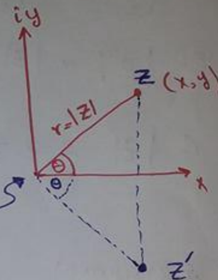
$x^2 + 9 = (x-3i)(x+3i)$

$$Z = x + iy = re^{i\theta} = r[\cos\theta + i\sin\theta]$$

$$e^{i\ldots} = \cos\ldots + i\sin\ldots$$

$Re(z) = x$

$Im(z) = y$



conjugate المرافق

$\bar{Z} = x - iy = re^{-i\theta}$

$\overline{(w + Z)} = \bar{w} + \bar{Z}$

$\overline{(wZ)} = \bar{w} \cdot \bar{Z}$

$\overline{\left(\frac{w}{Z}\right)} = \frac{\bar{w}}{\bar{Z}}$

modulus المقياس

هي بعد النقطة عن نقطة الأصل

$r = |Z| = \sqrt{x^2 + y^2}$

b [Properities]

1. $|z_1 + z_2| \leq |z_1| + |z_2|$
2. $|z_1 + z_2| \geq |z_1| - |z_2|$
3. $|z_1 z_2| = |z_1| \cdot |z_2|$
4. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$
5. $|z^n| = |z|^n$
6. $|z|^2 = z \cdot \bar{z}$

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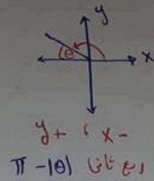
argument السعة

$$\theta = \arg(z) = \tan^{-1}(y/x)$$

هي الزاوية المحصورة بين النقطة ونقطة الأصل و محور السينات.

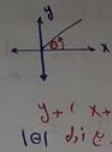
[Properities]

1. $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$
2. $\arg(z_1/z_2) = \arg(z_1) - \arg(z_2)$
3. $\arg(z)^n = n \arg(z)$



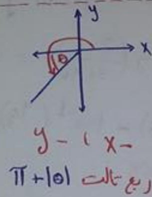
$y+$ ، $x-$

ربع ثاني $\pi - |\theta|$



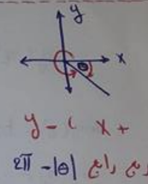
$y+$ ، $x+$

ربع أول $|\theta|$



$y-$ ، $x-$

ربع ثالث $\pi + |\theta|$



$y-$ ، $x+$

ربع رابع $2\pi - |\theta|$

adding & substracting

$$Z_1 \pm Z_2 = (x_1 \pm x_2) + i\,(y_1 \pm y_2)$$

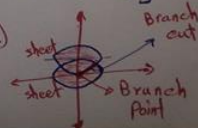
Roots of complex number

$$Z^{1/n} = r^{1/n}\left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i\sin\left(\frac{\theta + 2\pi k}{n}\right)\right]$$

$n = 2 \rightarrow$ جذرين

$K = (0, 1, 2, \ldots, n-1)$

$n \rightarrow$ تدل على عدد الجذور



d

sheet 1

~~1- find the real and imag~~

* Find modulus & argument of .

1) $Z = \frac{2+i}{3+4i}$

$$|z| = \left|\frac{2+i}{3+4i}\right| = \frac{|2+i|}{|3+4i|} = \frac{\sqrt{4+1}}{\sqrt{9+16}} = \frac{1}{\sqrt{5}}$$

$$\arg(z) = \arg\left(\frac{2+i}{3+4i}\right) = \arg(2+i) - \arg(3+4i)$$

ربع اول ربع اول

لن نغير قيم الزوايا

$$\arg(z) = \tan^{-1}(1/2) - \tan^{-1}(4/3)$$

$$\arg(z) = -26.565$$

2) $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$

$$|z| = \left|\frac{1+2i}{3-4i} + \frac{2-i}{5i}\right| = \left|\frac{5i(1+2i) + (3-4i)(2-i)}{(3-4i)(5i)}\right|$$

$$|z| = \left|\frac{5i-10+6-3i-8i-4}{15i+20}\right| = \left|\frac{-8-6i}{20+15i}\right|$$

$$|z| = \frac{|-8-6i|}{|20+15i|} = \frac{\sqrt{64+36}}{\sqrt{(20)^2+(15)^2}} = 0.4$$

(e)

$$\arg\left(\frac{-8-6i}{20+15i}\right) = \arg(-8-6i) - \arg(20+15i)$$

(ربع ثالث under $-8-6i$; ربع أول under $20+15i$)

$$\arg(z) = \left(\pi + \tan^{-1}\left(\frac{6}{8}\right)\right) - \tan^{-1}\left(\frac{15}{20}\right) = \pi$$

Find real & imagine part = Find polar form

$$z = x + iy = r\left[\cos\theta + i\sin\theta\right]$$

(a) $(1+\sqrt{3}\,i)^6$

$$r = |z| = \left|(1+\sqrt{3}i)^6\right| = \left|(1+\sqrt{3}i)\right|^6$$

$$= \left(\sqrt{1+3}\right)^6 = 2^6$$

$$\theta = \arg(z) = \arg(1+\sqrt{3}i)^6 = 6\arg(1+\sqrt{3}i)$$

(ربع أول)

$$= 6 * \tan^{-1}\frac{\sqrt{3}}{1} = 2\pi$$

so $z = 2^6\left[\cos 2\pi + i\sin 2\pi\right] = 2^6\left[1 + i0\right]$

$$z = 2^6 + 0i$$

2^6 → real part; $0i$ → Im part

F

b) $\left(\frac{1+i}{1-i}\right)^4$

$$r = |z| = \left|\frac{1+i}{1-i}\right|^4 = \left(\frac{|1+i|}{|1-i|}\right)^4 = \underline{1}$$

$$\theta = \arg(z) = \arg\left(\frac{1+i}{1-i}\right)^4 = 4\left[\arg(1+i) - \arg(1-i)\right]$$

$$\theta = 4\left[\tan^{-1}(1) - (2\pi - \tan^{-1}(1)\right] = \underline{-6\pi}$$

$$\therefore z = \cos(-6\pi) + i\sin(-6\pi)$$

$$z = 1 + 0i$$

---

2) show that

a) $1 + \cos(\theta) + \cos(2\theta) + \cdots \cos(n\theta)$

$$= \frac{1}{2} + \frac{\sin\left(n+\frac{1}{2}\right)\theta}{2\sin\left(\theta/2\right)}.$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\therefore \cos\theta = \mathrm{Re}\left(e^{i\theta}\right)$$

L.H.S $\longrightarrow \mathrm{Re}\left[1 + e^{i\theta} + e^{i2\theta} + \cdots + e^{in\theta}\right]$

المتوالية الهندسية

$$a + ar + ar^2 + \dots = a\,\frac{1-r^{n+1}}{1-r}$$

a ← العدد الأول

r ← الأساس

$$Re\left[\frac{1 - e^{i(n+1)\theta}}{1 - e^{i\theta}}\right] * \frac{e^{-i\frac{\theta}{2}}}{e^{-i\frac{\theta}{2}}}$$ بالضرب في

$$= Re\left[\frac{e^{-i\theta/2} - e^{i(n+\frac{1}{2})\theta}}{e^{-i\theta/2} - e^{i\theta/2}}\right]$$

$$= Re\left[\frac{\cos\left(\frac{-\theta}{2}\right) + i\sin\left(\frac{-\theta}{2}\right) - \cos(n+\frac{1}{2})\theta - i\sin(n+\frac{1}{2})\theta}{\cos\left(\frac{-\theta}{2}\right) + i\sin\left(\frac{-\theta}{2}\right) - \cos\left(\frac{\theta}{2}\right) - i\sin\left(\frac{\theta}{2}\right)}\right]$$

$$= Re\left[\frac{\cos\left(\frac{\theta}{2}\right) - i\sin\left(\frac{\theta}{2}\right) - \cos(n+\frac{1}{2})\theta - i\sin(n+\frac{1}{2})\theta}{-2i\sin\left(\frac{\theta}{2}\right)}\right]$$

$$= \frac{1}{2} + \frac{\sin(n+\frac{1}{2})\theta}{2\sin(\theta/2)} = R.H.S$$

#

[H] show that

[b] $$\frac{(\cos 2\theta - i \sin 2\theta)^7 (\cos 3\theta + i \sin 3\theta)^{-5}}{(\cos 4\theta + i \sin 4\theta)^{12} (\cos 15\theta - i \sin 5\theta)^{-6}}$$

$$= \cos 107\theta - i \sin 107\theta$$

sol.

$$L.H.S = \frac{(e^{-i2\theta})^7 (e^{i3\theta})^5}{(e^{i4\theta})^{12} (e^{-i5\theta})^{-6}}$$

$$= \frac{e^{-i14\theta} \cdot e^{-i15\theta}}{e^{i48\theta} \cdot e^{i30\theta}} = e^{i\theta[-14-15-48-30]}$$

$$= e^{-i107\theta} = \cos(107\theta) - \sin(107\theta) = R.H.S$$

[4] show that

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$$

$\because |z|^2 = z \cdot \bar{z}$

$$L.H.S \longrightarrow (z_1 + z_2)(\overline{z_1 + z_2}) + (z_1 - z_2)(\overline{z_1 - z_2})$$

$$= (z_1 + z_2)(\bar{z}_1 + \bar{z}_2) + (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$$

$$= z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2 + z_1\bar{z}_1 - z_1\bar{z}_2 - z_2\bar{z}_1 + z_2\bar{z}_2$$

$$= 2z_1\bar{z}_1 + 2z_2\bar{z}_2 = 2|z_1|^2 + 2|z_2|^2 = R.H.S$$

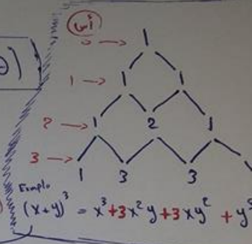
[5] Use De Moiver theorem to obtain

$\cos 3\theta$ & $\frac{\sin 3\theta}{\sin\theta}$ in terms of power of $\cos\theta$.

[I]

De moiver theorem

$$(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)$$



Example $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

at $n = 3$

$$(\cos\theta + i\sin\theta)^3 = (\cos 3\theta + i\sin 3\theta)$$

$$= (\cos\theta)^3 + 3(\cos\theta)^2(i\sin\theta) + 3(\cos\theta)(i\sin\theta)^2 + (i\sin\theta)^3$$

$$= \cos^3\theta + 3i\cos^2\theta\sin\theta + -3\cos\theta\sin^2\theta - i\sin^3\theta$$

$$= \cos 3\theta + i\sin 3\theta$$

Real Part

$$\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$$

$$\cos 3\theta = \cos^3\theta - 3\cos\theta\cdot[1 - \cos^2\theta]$$

Im Part

$$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$$

$$\frac{\sin 3\theta}{\sin\theta} = 3\cos^2\theta - (1 - \cos^2\theta)$$